Simulating Ocean Water

海水模拟

1. These notes are intended to give computer graphics programmers and artists an introduction to methods of simulating, animating, and rendering ocean water environments. CG water has become a common tool in visual effects work at all levels of computer graphics, from print media to feature films. Several commercial products are now available for nearly any computer platform and work environment, in addition to proprietary tool held by a few companies, which generate high quality geometry and images. In order for an artist to exploit the tool to maximum benefit, it is important that he or she become familiar with concepts, terminology, a little oceanography, and the present state of the art.

这篇文章旨在为计算机图形学编程人员和美术人员介绍一些模拟渲染海洋环境的方法。计算机图形学中，水已经成为一种重要的视觉表现效果。不管在传统纸质媒体或者事现代大片中都又较大的应用。目前，已经有了相关商业性的软件产品。有些公司甚至开发出了能够高质量的生成相关的几何物体和图片的软件。而对于美术人员来说，了解一些相关的技术，概念，海洋学的知识无疑可以使她们在工作上如虎添翼。

As demonstrated by the pioneering efforts in the films Water-world and Titanic, as well as several other films made since about 1995, images of cg water can be generated with degree of realism. However, this level of realism is limited to relatively calm, nice ocean conditions. Conditions with large amounts of spray, breaking waves, foam, splashing, and wakes are approaching the same realistic look.

早在1995年，电影《Water-world》和《Titanic 》就已经使用了图形学中相关的技术来模拟海水，并且得到了不错的模拟效果。然而，这样的模拟效果只有当海洋在相对来说比较平静时才足够的逼真。当需要模拟有大量的喷雾，断裂的海浪，海水飞溅和泡沫等情况时，目前的模拟效果虽然还不是特别逼真，但是相关的技术正在逐渐完善。

Broadly, the reader should come away from this material with (1) an understanding of some algorithms that generate/animate water surface height fields suitable for modeling waves as big as storm surges and as small as tiny capillaries; (2) an understanding of basic optical processes of reflection and refraction from a water surface; (3) an introduction to the color filtering behavior of ocean water; (4) an introduction to complex lighting effects known as caustic and godrays, produced when sunlight passes through the rough surface into the water volume underneath; and (5) some rules of thumb for which choices make nice looking images and what are the tradeoffs of quality versus computational resources. Some example shaders are provided, and example renderings demonstrate the content of the discussion.

总的来说，通过阅读本文，读者可以了解到以下知识。（1）生成适合模拟大到暴风卷起的海浪，小到小浪花的网格模型。（2）对于水平面的光学反射和折射的基本处理过程和方法。（3）海水颜色过滤。（4）一些比较复杂的光照效果，比如在太阳光穿透海水表面光线直抵海水深处时产生的焦散和上帝之光。（5）当计算资源和模拟效果的需要权衡的时候的处理原则。在本文中，我给出了一些与所探讨的内容紧密相连的例子的和其shader实现方式。

Before diving into it, I first want to be more concrete about what aspect of the ocean environment we cover ( or not cover) in these notes. Figure 1 is a rendering of oceanscape produced from models of water, air, and clouds. Light from the clouds is reflected from the surface. On the extreme left, sun glitter is also present. The generally bluish color of the water is due to the reflection of blue skylight, and to light coming out of the water after scattering from the volume. Although these notes do not tackle the modeling and rendering of clouds and air, there is a discussion of how skylight from clouds and air is reflected from, or refracted through, the water surface. These notes will tell you how to make a height-field displacement-mapped surface for the ocean waves with the detail and quality shown in the figure. The notes also discuss several effects of the underwater environment and how to model/render them. The primary four effects are sunbeams ( also called god rays), caustics on underwater surfaces, blurring by the scattering of light, and color filtering.

在进入正文之前，我想再进一步明确一下本文将涉及（或者不涉及）海洋环境模拟的那些方面。图1时衣服综合了海水，空气，云层的海洋环境模拟渲染结果。其中，穿过云层的光线在海水平面上会发生反射。在图片最左边的地方，也可以看见太阳过不穿过云层直接照射海平面的体现。海水的蓝色主要又海水对天空到来的光线的散射效果决定的。本文中我们不会讨论云层和天空模型的模拟，但是我们会讨论通过它们来到海平面中发生的反射和折射现象。本文我们会给出一种使用高度图来表示海水平面各个地方高度的方法。还将讨论几种影响模拟海平面以下的海水的渲染效果的因素，最主要的四种是：上帝之光，海平面以下的焦散，模糊光散射和颜色过滤。

There are also many other complex and interesting aspects of the ocean environment that will not be covered. These include breaking waves, spray, foam, wakes around objects in the water, splashes from bodies that impact the surface, and global illumination of the entire ocean-atmosphere environment. There is substantial research underway on these topics, and so it is possible that future versions of this or other lecture notes will include them. I have included a brief section on advanced modifications to the basic wave height algorithm that produce choppy waves. the modification could feasibly lead to a complete description of the surface portion of breaking waves, and possibly serve to drive the spray and foam dynamics as well.

但也确实有一些复杂而又有趣的方面我们没有涉及到，包括波浪断裂，海水飞溅，泡沫，海水中有物体时的海浪，海水中的气泡对海表面的影响以及整个海洋大气的全局关照。这些也是该话题下所要讨论的重要内容，所以我可能会在本文以后的版本中或者在另外一个文章中将这些因素包括进来。在本文中，有一小节中我给出了一种在对基本波浪高度算法改进后能够适用于巨浪的方法。进一步地改进甚至能够得到模拟波浪断裂，海水喷雾和海水泡馍的模拟效果。

There is, of course, a substantial body of literature on ocean surface simulation and animation, both in computer graphics circles and in oceanography. One of the first descriptions of water waves in computer graphics was by Fournier and Reeves, who modeled a shoreline with waves coming up on it using as water surface model called Gerstner waves. In that same issue, Darwin Peachey presented a variation on this approach using basis shapes other that sinusoids.

当然，关于海水表面模拟的文献有很多，它们有的是计算机图形学学者提出的，有的时海洋学学者提出的。其中有一个波浪模拟的方法，由计算机图形学学者Fournier和Reeves提出。它们使用了Gerstner波的方法模拟了一个海岸线上的海浪的起伏。在同样的问题中，Darwin Peacher使用了同一种方法来模拟，但是它们没有选择正线函数，而是选择了其他的基本函数形式。

In the oceanographic literature, ocean optics became an intensive topic of research in the 1940s. S.Q. Duntley published in 1963 papers containing optical data of relevance to computer graphics. Work continues today. The field of optical oceanography has grown into a mature quantitative science with sub disciplines and many different applications. One excellent review of the state of the science was written by Curtis Mobley.

在海洋学文献中，海洋光学在20世纪40年代成为一个研究热点。S.Q.Duntley在1963年发表了一片综合了计算机图形学和海洋光学的论文中。该领域的研究一直持续到今天。现在，海洋光学已经发展成为了一个能够定量分析，且具有二级学科，有许多应用领域的学科。关于这门学科的知识可以参考在Curtis Mobley的文章。

In this lectures the approach we take to creating surface waves is close to the one outlined by Masten, Watterberg, and Marred, although the technique had been in use for many years prior to their paper in the optical oceanography community. This approach synthesizes a patch of ocean waves from a Fast Fourier Transform(FFT) prescription, with user-controllable size and resolution, and which can be tiled seamlessly over al larger domain. The patch contains many octaves of sinusoidal waves that all add up at each point to produce the synthesize height. The mixture of sinusoidal amplitudes and phases however, comes from statistical, emperically based models of the ocean. What makes these sinusoids look like waves and not just a bunch of sine waves is the large collocation of their animation using they dispersion relation. We examine the impact of the number of sinusoids and resolution on the quality of the render image.

在本文中我们使用的产生海平面和波浪的方法与Masten, Watterberg和Marred提出的方法类似，该技术在海洋光学中已经使用了很多年了。这样发放的基本思想是：使用快速傅立叶变换得到若干个由正弦波组成的波浪分量，再将这些波浪分量根据一定的策略在某点综合叠加起来，从而得到再该点处最终的海平面高度。在综合各波浪分量叠加是，它们的正弦波振幅和相位的叠加关系是通过统计真实的海洋运动数据得来的，既这是一种经验模型。基于什么样的策略使我们能够使用正弦波来模拟波浪呢？这与他们的色散关系有关。我们需要关注用户模拟的正弦波的数量对渲染效果的影响。

In the next section we begin the discussion of the ocean environment with a broad introduction to the global illumination problem. The radiosity equations for this environment look much like those of any other radiosity problem, although the volumetric character of some of the environmental components complicate a general implementation considerably. However, we simplify the issues by ignoring some interactions and replacing others with models generated by remote sensing data.

在下一节中，我们将开始讲解在海洋环境中全局光照技术的使用。在这样的环境下，辐射度方程与其他情况下类似。在这里，为了简化计算，我们通过忽略一些物体的相互作用并使用遥感数据分析得到的模型来进行替代，从而去除了海水体积因素给光照计算带来的复杂性。

Practical methods are presented in section 3 for creating realizations of ocean surfaces. We present two methods, one based a simple model of water structure and movement, and one based on summing up large numbers of sine waves with amplitudes that are related to each other based on experimental evidence. This second method carries out the sum using the technique of Fast Fourier Transformation (fft), and has been used effectively in projects for commercials, television, and motion pictures.

第三节中我们将介绍两种可用于模拟海水表面网格波浪且实用性方法。一种是基于简单模型的海浪结构和移动，另外一种是基于若干正弦波按照一定策略叠加，并基于一些经验模型的方法。后者使用了快速傅立叶变换计算叠加结果，在很多项目和商业应用，电视电影中都已经被广泛使用。

After the discussion of the structure and animation of the water surface, we focus on the optical properties of water relevant to the graphic problem. First, we discuss the interaction at the air-water interface: reflection and refraction. This leaves us with a simple but effective Renderman-style shader suitable for rendering water surfaces in BMRT, for example. Next, the optical characteristics of the underwater environment are explored.

在讨论了海水表面的结构和海浪模拟问题以后，我们将讨论海水的光学属性，这与计算机图形学息息相关。首先我们讨论海水-空气接触面上发生的反射和折射问题。我们可以使用简单而又高效且符合图像渲染规范的shader来渲染海水表main。然后我们讨论光学属性对于海平面以下的海水的影响。

1. In this section we focus on algorithms and practical steps to building height fields for ocean waves. Although we will be occupied mostly by a method based on Fast Fourier Transform (FFTs), we begin by introducing a simpler description called Gerstner Waves. This is a good starting point for several reasons: the mathematics is relatively light compared to FFTs, several important oceanographic concepts can be introduced, and they give us a chance to discuss wave animation. After this discussion of Gerstner waves, we go after there complex FFT method, which produces wave height fields that are more realistic. These waves, called “linear waves” or “gravity waves” are a daily realistic representation of typical waves on the ocean when the weather is not too stormy. Linear waves are certainly not the whole story, and so we discuss also some methods by which oceanographers expand the description to “nonlinear waves”, waves passing over a shallow bottom, and very tiny waves about one millimeter across called capillary waves.

在本节中，我们将关注生成波浪高度图的算法和实际的步骤。虽然我们主要使用的是基于快速傅立叶变换的方法，但是我们将先介绍一种比较简单的方法叫做Gerstner波。介绍这种方法有以下的几点好处：数学计算相较于基于傅立叶变换的方法要更简单，能够由此介绍几种海洋图形学相关的概念，能够介绍波波浪动画的相关原理。在讲解了Gerstner波以后，我们将介绍更加复杂的基于傅立叶变换的方法，这种方法所得到的波浪的高度域更佳符合实际的结果。这种波，称为“线性波”或者“重力波“，在较平静的海水中有比较好的模拟结果。现行波当然不是我们讨论的终点，我们还会在此基础上继续拓展，继而介绍”非线性波“，它能够沿着浅水底部传播。波长只有约1毫米的非线性波称为微小波。

In the course of this discussion, we will see how quantities like windspeed, surface tension, and gravitational acceleration come into the practical implementation of the algorithms.

在本节的讨论中，我们将会看到风速，海水网格密度和重力加速度是应用在算法当中的。

Gerstner Waves

Gerstner waves were first found as an approximate solution to the fluid dynamic equations almost 200 years ago. There first appli- cation in computer graphics seems to be the work by Fournier and Reeves in 1986 (cited previously). The physical model is to describe the surface in terms of the motion of individual points on the surface. To a good approximation, points on the surface of the water go through a circular motion as a wave passes by. If a point on the undisturbed surface is labelled x0 = (x0,z0) and the undisturbed height is y0 = 0, then as a single wave with amplitude A passes by, the point on the surface is displaced at time t to

x  = x0 −(k/k)Asin(k·x0 −ωt) (9)

y  = Acos(k·x0 −ωt). (10)

In these expressions, the vector k, called the wavevector, is a horizontal vector that points in the direction of travel of the wave, and has magnitude k related to the length of the wave (λ) by

k = 2π/λ (11)

The frequency w is related to the wave vector, as discussed later. Figure 3 shows two example wave profiles, each with a different value of the dimensionless amplitude kA. For values kA < 1, the wave is periodic and shows a steepening at the tops of the waves as kA approaches 1. For kA > 1, a loop forms at the tops of the wave, and the “insides of the wave surface are outside”, not a particularly desirable or realistic effect.

As presented so far, Gerstner waves are rather limited because they are a single sine wave horizontally and vertically. However, this can be generalized to a more complex profile by summing a set of sine waves. One picks a set of wave vectors ki, amplitudes Ai, frequencies ωi, and phases φi, for i = 1,...,N, to get the expressions

x = x0 − (ki/ki)Ai sin(ki · x0 − ωit + φi) (12)

Gerstner波在两百年起就已经作为模拟流体运动的近似方法了。1986年，Fournier和Reeves第一次将这种方法应用在计算机图形学中。其物理模型是海水表面有一系列点组成，通过改变海水表面的点的坐标使其作曲线运动来模拟海浪的产生。如果海水表面一点在完全平静时的坐标，记x0 = (x0,z0)且y0 = 0，那么它的变换方式是只有一个单一的波，变换方式为

x  = x0 −(k/k)Asin(k·x0 −ωt) (9)

y  = Acos(k·x0 −ωt). (10)

在这个公式中，向量k叫做波向量，是一个水平方向的向量，向量的方向就是波浪传播的方向。k与波长λ有关

k = 2π/λ (11)

频率w与波向量k有关，稍后将会详细讨论。

图3展示了两个波的实例，他们的振幅A不同。如果kA < 1，波是具有周期性的，并且kA的越接近1，那么波在顶部的时候回越陡峭。如果kA > 1，那么在波的顶部会出先翻转，即海水内表面变成外表面，外表面变成内表面，这是不符合现实的。

可以看出，Gerstner波的限制是比较多的，因为在水平方向和竖直方向只有一个正弦波。然而，通过将正弦波叠加，可以的到很多复杂的模型。对于每一个波向量ki, 振幅Ai, 频率ωi, 相位φi, i 取1,...,N，我们可以得到

x = x0 − (12)

y  = (10)

图四是有四个波叠加的结果。还有很多由于的波浪形状都可以通过这种方式得到。

3.2波的动效：色散关系

The aminated behavior of Gerstner waves is determined by the set of frequencies ωi chosen for each component. For water waves, there is a well-known relationship between these frequencies and the magnitude of their corresponding wavevectors, ki. In deep water, where the bottom my be ignored, that relationship is

ω2(k) = gk (14)

The parameter g is the gravitational constant, nominally 9.8/sec­2. This dispersion relationship holds for Gerstner waves, and also for the FFT-based introducted next.

对于Gerstner而言，起运动关系是有一系列的ωi决定的。对于水产生的波来说，在平率和波向量之间有一个著名的关系。如果水很深，那么水底的影响可以忽略，其关系为：

ω2(k) = gk (14)

g是重力常亮，通常取9.8m/sec2。这个关系式不仅对于Gerstner成立，对于下一届要讲的基于傅里叶变换的方法也同样适用。

There are several conditions in which the dispersion relationship is modified. When the bottom is relatively shallow compared to the length of the waves, the bottom jas a retarding affect on the waves. For a bottom at a depth D below the mean water level the dispersion relation is

ω2(k) = gk tanh(kD) (15)

Notice that if the bottom is very deep, the behavior of the tanh function reduces this dispersion relation to the previous one.

在几种情况下，色散关系要做一些修改。若海水深度相对于波长来水比较小时，底部的粘滞力会对波浪有所影响。若海水深度为D，那么色散关系为

ω2(k) = gk tanh(kD) (15)

注意到如果水非常浅，那么tanh函数的值将为1

A second situation which modifies the dispersion relation is surface tension. Very small waves, with a wavelength of about 1 cm or less, have an addition term:

ω2(k) = gk(1 + k2L2) , (16)

and the parameter L has units of length. Its magnitude is the scale for the surface tension to have effect.

第二种情况的影响因素是网格表面的密度。对于比较小的波，如果波长为1cm或者以下，那么色散关系为

ω2(k) = gk(1 + k2L2) , (16)

其中L取非负整数，它的大小是与网格密度有关。

Using these dispersion relationships, it is very difficult to create a sequence of frames of water surface which for a continuous loop. In order to have the sequence repeat after a certain amount of time T for example, it is necessary that all frequencies be multiple of the basis frequence

ω0 ≡ (17)

However, when the wavevectors k are distributed on a regular lattice, it is impossible to arrange the dispersion-generated frequencies to also be on a uniform lattice with spacing ω0

如果使用这样的色散关系，那么很难是的海水的运动能够具有周期性。为了满足这样的周期性，周期为T，那么有必要让每一个频率都乘上基频率

ω0 ≡ (17)

然而，如果波向量k是规则分布的，那么我们没有办法色散关系满足频率都按ω0分布。

The solution to that is to not use the dispersion frequencies, but instead a set that is close to them. For a given wavenumber k, we use the frequence

ω ̄(k) = (18)

where [[a]] means take the integer part of the value of a, and ω(k) is any dispersion relationship of interest. The frequencies ω ̄(k) are a quantization of the dispersion surface, and the animation of the water surface loops after af time T because the quantized frequencies are all integer multiples of ω0. Figure 5 plots the original dispersion curve, along with quantized dispersion curves for two choices of the repeat time T.

解决这个问题的办法是不适用色散频率，而是使用一个接近色散频率的值。对于某个波k，我们使用频率

ω ̄(k) = (18)

其中[[a]]表示对a取整，ω(k)是给定的色散关系。频率ω ̄(k)是对色散表面的量化，按照这个关系的运动是满足周期为T的，因为他们都乘了ω0，图5展示了使用原始色散关系和量化后的色散关系的两种结果。

3.3 统计波模型和傅里叶变换

Oceanographic literature tends to downplay Gerstner waves as realistic model of the ocean. Instead, statistical models are used, in combination with experimental observations. In the statistical models, the wave height is considered a random variable of horizontal position and time, h(x,t).

海洋图形学文献中一班认为Gerstner波的效果并不好，从实际观察结果来看，他们认为统计模型的效果比较好。在统计模型中，波的高度是一个关于水平面坐标的随机变量，h(x,t)。

Statistical models are also based on the ability to decompose the wave height field as a sum of sine and cosine waves. The value of this decomposition is that the amplitudes of the waves have nice mathematical and statistical properties, making it simpler to build models. Computationally, the decomposition uses Fast Fourier Transforms (ffts), which are a rapid method of evaluating the sums.

统计模型基于把高度域分解为一系列正弦和余弦波的叠加。但是每一个波的振幅有良好的数学和统计属性，使得他们建立模型更加简单。在计算上，这种分解也是使用了快速傅里叶变化。快速傅里叶变换是一种快熟计算和的方法。

The fft-based representation of a wave height field expresses the wave height h(x, t) at the horizontal position x = (x, z) as the sum of sinusoids with complex, time-dependent amplitudes:

h(x,t) = h(k,t) exp(ik · x) k (19)

where t is the time and k is a two-dimensional vector with components k = (kx, kz), kx = 2πn/Lx, kz = 2πm/Lz, and n and m are integers with bounds −N/2 ≤ n < N/2 and −M/2 ≤ m < M/2. The fft process generates the height field at discrete points x = (nLx/N,mLz/M). The value at other points can also be obtained by switching to a discrete fourier transform, but under many circumstances this is unnecessary and is not applied here. The height amplitude Fourier components, h ̃(k,t), determine the structure of the surface. The remainder of this subsection is concerned with generating random sets of amplitudes in a way that is consistent with oceanographic phenomenology.

基于fft的方法将波浪的高度域看做是水平面坐标x = (x, z)上的有若干与时间有关的正弦波叠加得到的。

h(x,t) = h(k,t) exp(ik · x) k (19)

其中t是时间，k是一个二维向量，k = (kx, kz), kx = 2πn/Lx, kz = 2πm/Lz, n和m都是整数，并且满足−N/2 ≤ n < N/2 and −M/2 ≤ m < M/2。在其他点处的的高度只也可以通过离散傅里叶变换来计算，但是通常是没有必要的，所以我们这里没有介绍。式中h ̃(k,t)决定了海水表面结构。剩下的问题就是要解决如何生成随机振幅值使得模拟的结果接近真实情况。